Dimensional Analysis of Drop Problem

$$H = f^{\underline{n}}(a, g, \rho_d, \rho_f, \mu_d, \mu_f)$$

$$\begin{array}{c|ccccccc} & H & a & g & \rho_d & \rho_f & \mu_d & \mu_f \\ \hline M & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ L & 1 & 1 & 1 & -3 & -3 & -1 & -1 \\ T & 0 & 0 & -2 & 0 & 0 & -1 & -1 \end{array}$$

full rank, so $7-3=4$ groups

$$\frac{H}{a}, \quad \frac{\rho_d}{\rho_f}, \quad \frac{\mu_d}{\mu_f}, \quad Re$$

$$Re = \frac{(\rho_d - \rho_f) g a^2}{\mu} \frac{a \rho_f}{\mu} = \frac{\Delta\rho g a^3 \rho_f}{\mu^2}$$

Strengthen:

Now we suppose that $\rho_d - \rho_f \ll \rho_f$

So $\Delta\rho$ only comes into gravitational term!

$$\therefore H = f^{\underline{n}}(a, \Delta\rho g, \rho_f, \mu_d, \mu_f)$$

$$\begin{array}{c|cccccc} & H & a & \Delta\rho g & \rho_f & \mu_d & \mu_f \\ \hline M & 0 & 0 & 1 & 1 & 1 & 1 \\ L & 1 & 1 & -2 & -3 & -1 & -1 \\ T & 0 & 0 & -2 & 0 & -1 & -1 \end{array}$$

still of full rank, $\therefore$ $6-3=3$

$$\therefore \frac{H}{a} = f^{n}\left(\frac{\mu_d}{\mu_f}, Re\right)$$

$$Re = \frac{\Delta \rho g a^3 \rho_f}{\mu^2}$$

So if we fix μ_d/μ_f. (choose same materials) we can determine how H/a varies from a few experiments.

Empirically, we can determine the distance to breakup, normalized by a, for different drop sizes.

We can also look at different composition drop fluids, varying μ_d/μ_f, to see how that affects process.